**Unit 2 – Reading 2**

**Motion and Trajectories**

**Two Representations of Motion**

We now have two different ways of representing constant velocity using mathematical equations which look different from one another. On the one hand, we have the next-x functions which we have been using to create our simulations. On the other, we have an equation for position as a function of time. We can call these two representations the ‘position-based’ equation and ‘time-based’ equation.

|  |  |
| --- | --- |
| **Position-Based** | **Time-Based** |
|  |  |

The obvious question is: why do we these two different representations? In truth, while both of these equations describe the same phenomenon of motion of an object with constant velocity, they represent two different ideas in physics. The first is what we call a ***dynamical law*** while the second is a ***trajectory***.

**Dynamical Laws**

A dynamical law in physics is general rule which objects must obey. They are ***generative***, meaning they are responsible for producing the behaviors we observe. This is why we use dynamical laws in our simulations; they tell all of the objects in the simulation how they are supposed to behave.

One very important features of dynamical laws which we will see later is that even if we know that all objects must follow them, we do not necessarily know what behavior will result from them. Weather is a good example of this. The dynamical laws which dictate the behavior of weather patterns are reasonable well understood, but that does not mean we know the exact path that a storm will follow or how much it will rain on a given night.

In order to know exactly how an object will behave, we must know both the dynamical laws which govern its behavior and its ***initial conditions***. In classical physics, if we know both of these with extreme precision, we can predict the behavior of just about any macroscopic system. A pencil balanced on its tip is a good example of this. The rules governing how that pencil will fall are very simple, but in order to predict which direction the pencil will fall would require an extremely high level of knowledge of its initial conditions.

**Trajectories**

A trajectory is the path that an object follows written as a function of time. They are ***descriptive***, meaning they tell us exactly where that object will be at any point at time. They tell us how objects behave but tell us nothing about why the objects behave that way. Unlike dynamical laws, however, trajectories do not constrain the behavior of objects or systems. They do not tell the object how to behave, and instead serve as a record of where the object was and when it was there. Trajectories are specific solutions to the dynamical laws with certain initial conditions.

An important thing to keep in mind is that not all trajectories that we think of are physically possible given the dynamical laws of our universe. For example, one dynamical law we know is that no objects with mass can have a speed greater than the speed of light. I can write an infinite number of trajectories for objects which violate that law, but we will never observe an object moving with any of them.

**The Interplay Between Laws and Trajectories**

As physicists, our primary concern is uncovering and understanding the dynamical laws which govern our universe. We don’t just want to describe *how* objects behave, but to understand *why*. Unfortunately, dynamical laws are not observable by themselves, so to learn about the dynamical laws of the universe physicists try to find patterns in the trajectories of objects. From these patterns, we make guesses about what the laws might be. These guesses allow us to make predictions about other trajectories we might see given certain initial conditions. If we can confirm these predictions, our confidence in the supposed laws increases. If our predictions don’t match our observations, then we go back to the drawing board and hypothesize different laws to explain our observations.

Another reason to consider both trajectories and dynamical laws is that some trajectories which can be explained simply using words are exceedingly difficult to describe mathematically. Take for example a ball dropped from some initial height. Most people from experience could describe in words how the ball will move, but how could the trajectory of the ball be described mathematically? What equation would fit the graph of the ball’s trajectory shown below?

While this trajectory is difficult to represent mathematically, the motion itself is fairly easy to reproduce once the dynamical laws which govern the motion of falling objects are understood.